Name	Class

MATHS TEACHER HUB

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Vectors

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

•Use **black** ink or ball-point pen.
•**Fill in the boxes** at the top of this page with your name, centre number and candidate number.
•Answer **all** questions.
•Answer the questions in the spaces provided
– there may be more space than you need.
•Diagrams are NOT accurately drawn, unless otherwise indicated.
•You must **show all your working out.**
•If the question is a **1H** question you are not allowed to use a calculator.
•If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

Information

•The marks for **each** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

•Read each question carefully before you start to answer it.
•Keep an eye on the time.
•Try to answer every question.
•Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

6 $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

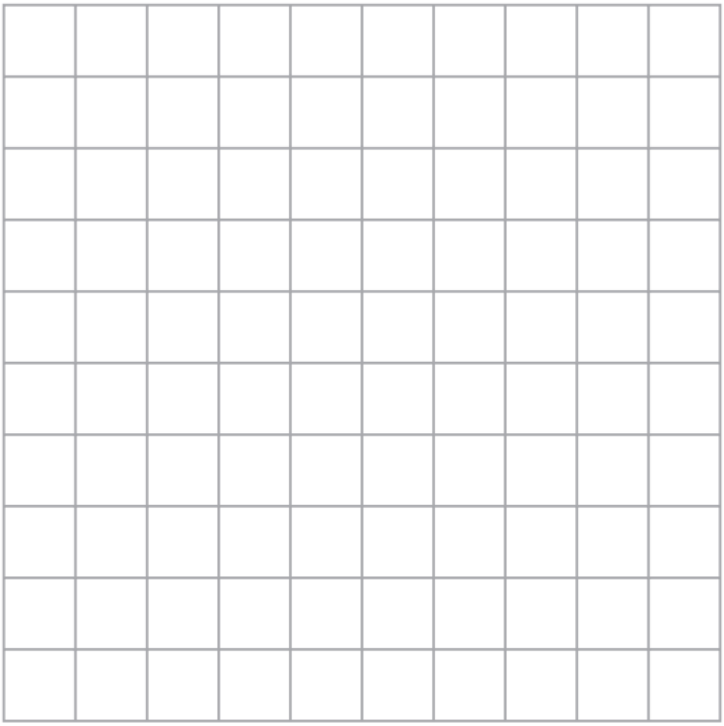
Find $2\mathbf{a} - 3\mathbf{b}$ as a column vector.

$\begin{pmatrix} \ldots\ldots\ldots \\ \ldots\ldots\ldots \end{pmatrix}$

(Total for Question 6 is 2 marks)

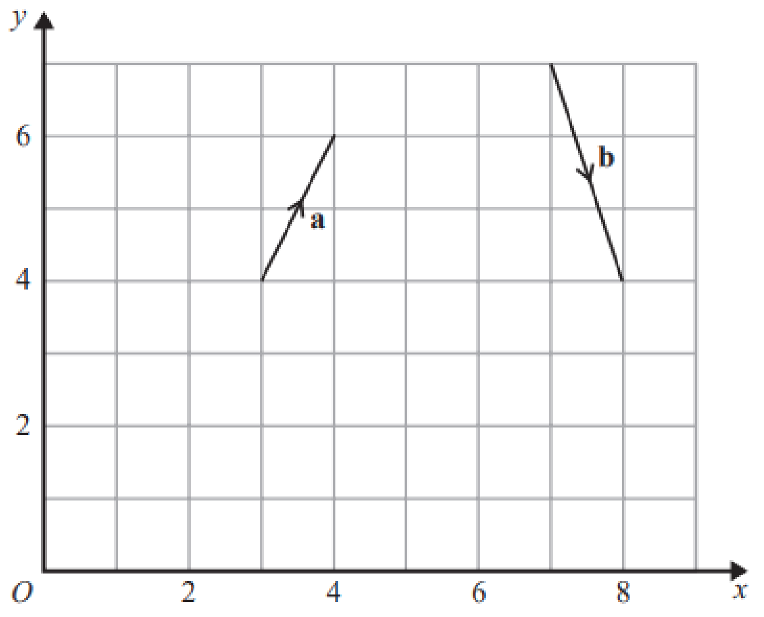
8 $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

On the grid below, draw and label the vector $2\mathbf{a} + \mathbf{b}$



(Total for Question 8 is 3 marks)

10 The vector **a** and the vector **b** are shown on the grid.



(a) On the grid, draw and label vector –2**a**

(1)

(b) Work out **a** + 2**b** as a column vector.

$\begin{pmatrix} \dots\dots \\ \dots\dots \end{pmatrix}$

(2)

(Total for Question 10 is 3 marks)

13 **a** and **b** are vectors such that

$$\mathbf{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \qquad \text{and} \qquad 3\mathbf{a} - 2\mathbf{b} = \begin{pmatrix} 8 \\ -17 \end{pmatrix}$$

Find **b** as a column vector.

$$\begin{pmatrix} \dots\dots\dots \\ \dots\dots\dots \end{pmatrix}$$

(Total for Question 13 is 3 marks)

15 A, B and C are three points such that

$$\overrightarrow{AB} = 3\mathbf{a} + 4\mathbf{b}$$

$$\overrightarrow{AC} = 15\mathbf{a} + 20\mathbf{b}$$

(a) Prove that A, B and C lie on a straight line.

(2)

D, E and F are three points on a straight line such that

$$\overrightarrow{DE} = 3\mathbf{e} + 6\mathbf{f}$$

$$\overrightarrow{EF} = -10.5\mathbf{e} - 21\mathbf{f}$$

(b) Find the ratio

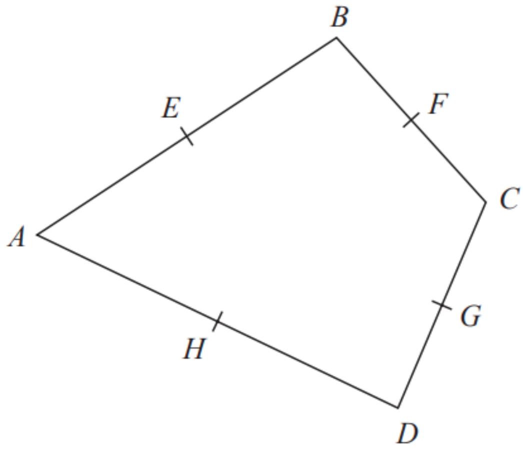
length of DF : length of DE

....

(3)

(Total for Question 15 is 5 marks)

18



ABCD is a quadrilateral.

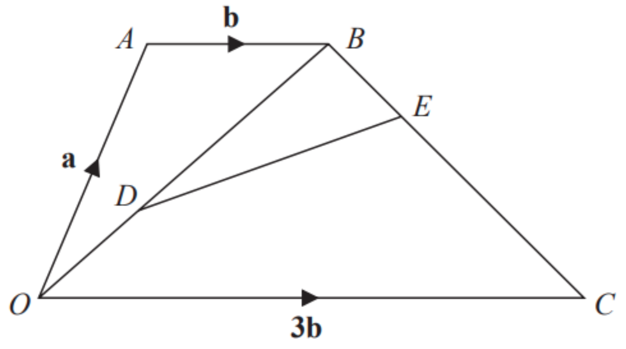
E, *F*, *G* and *H* are the midpoints of *AB*, *BC*, *CD* and *DA*.

$$\overrightarrow{AH} = \mathbf{a} \qquad \overrightarrow{AE} = \mathbf{b} \qquad \overrightarrow{DG} = \mathbf{c}$$

Prove, using vectors, that *EFGH* is a parallelogram.

(Total for Question 18 is 4 marks)

18 *OABC* is a trapezium.



$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{AB} = \mathbf{b}$
$\overrightarrow{OC} = \mathbf{3b}$

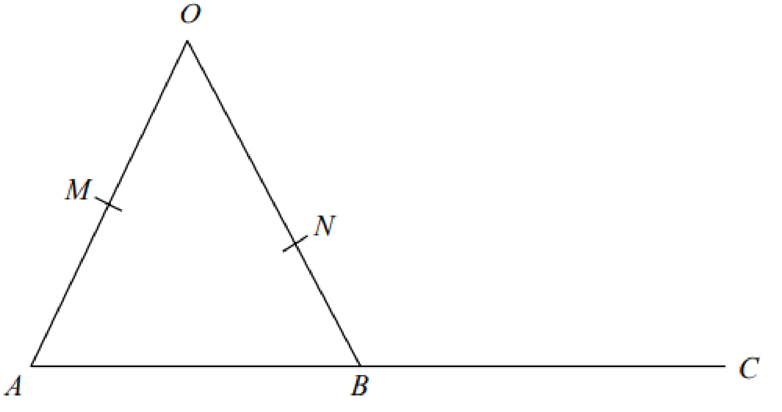
D is the point on *OB* such that $OD : DB = 2 : 3$
E is the point on *BC* such that $BE : EC = 1 : 4$

Work out the vector $\overrightarrow{DE}$ in terms of **a** and **b**.
Give your answer in its simplest form.

....

(Total for Question 18 is 4 marks)

18

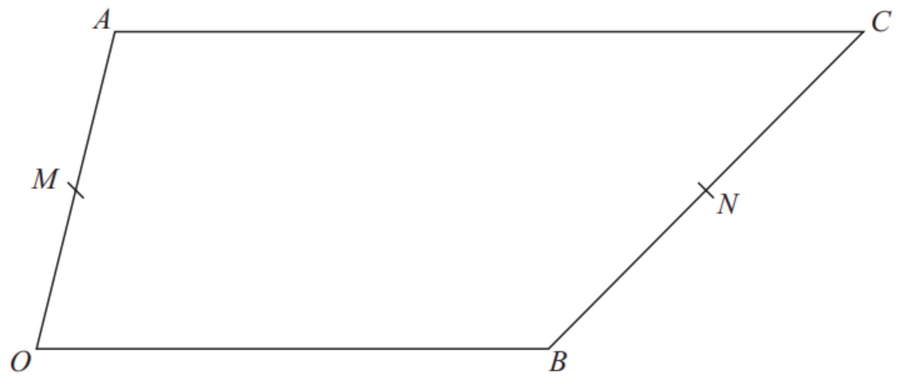


OMA, *ONB* and *ABC* are straight lines.
M is the midpoint of *OA*.
B is the midpoint of *AC*.
$\overrightarrow{OA} = 6\mathbf{a}$ $\quad \overrightarrow{OB} = 6\mathbf{b}$ $\quad \overrightarrow{ON} = k\mathbf{b}$ where k is a scalar quantity.

Given that *MNC* is a straight line, find the value of k.

(Total for Question 18 is 5 marks)

19 The diagram shows quadrilateral $OACB$.



M is the midpoint of OA.
N is the point on BC such that $BN : NC = 4 : 5$

$\overrightarrow{OA} = \mathbf{a}$ $\quad \overrightarrow{OB} = \mathbf{b}$ $\quad \overrightarrow{AC} = k\mathbf{b}$ where k is a positive integer.

(a) Express $\overrightarrow{MN}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

....

(4)

(b) Is MN parallel to OB?
Give a reason for your answer.

....

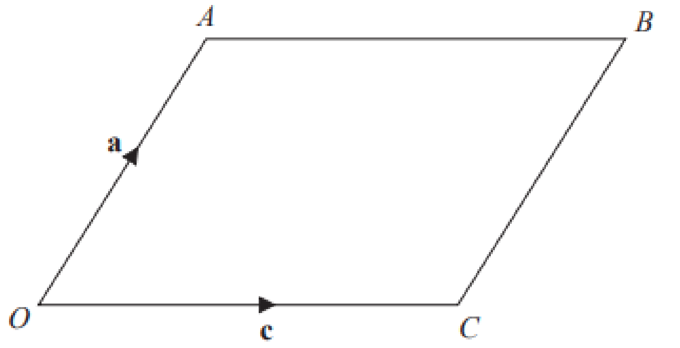
....

....

(1)

(Total for Question 19 is 5 marks)

19



$OABC$ is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

X is the midpoint of the line AC.
OCD is a straight line so that $OC : CD = k : 1$

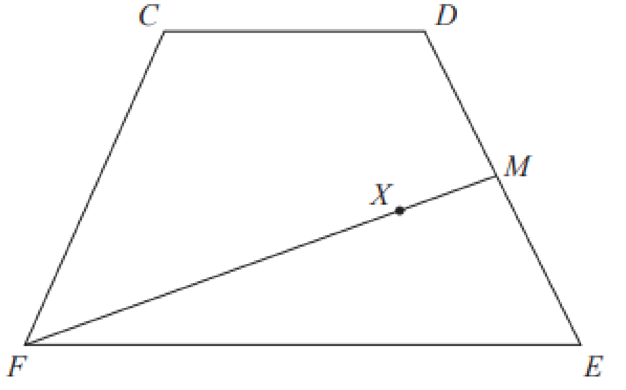
Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$

find the value of k.

$k =$ ....

(Total for Question 19 is 4 marks)

20 *CDEF* is a quadrilateral.



$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{FE}$ in terms of **a** and/or **b**.
Give your answer in its simplest form.

....
(2)

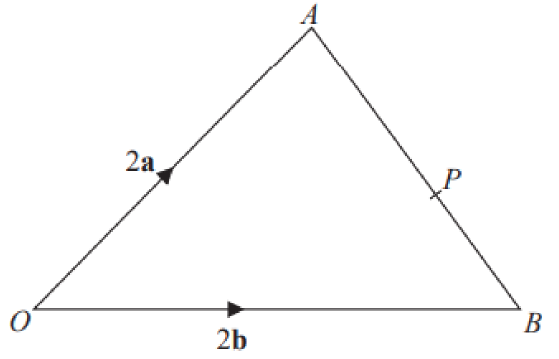
M is the midpoint of *DE*.
X is the point on *FM* such that $FX : XM = n : 1$
CXE is a straight line.

(b) Work out the value of *n*.

$n =$ ....
(4)

(Total for Question 20 is 6 marks)

20



OAB is a triangle.
P is the point on *AB* such that $AP : PB = 5 : 3$

$\overrightarrow{OA} = 2\mathbf{a}$

$\overrightarrow{OB} = 2\mathbf{b}$

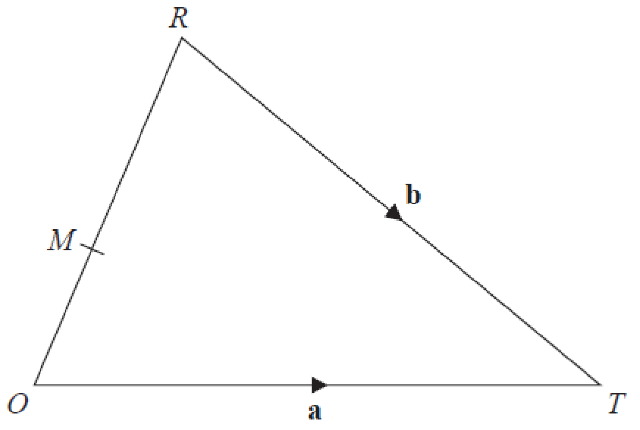
$\overrightarrow{OP} = k(3\mathbf{a} + 5\mathbf{b})$ where k is a scalar quantity.

Find the value of k.

....

(Total for Question 20 is 4 marks)

20 *ORT* is a triangle.



$\overrightarrow{OT} = \mathbf{a}$ $\overrightarrow{RT} = \mathbf{b}$

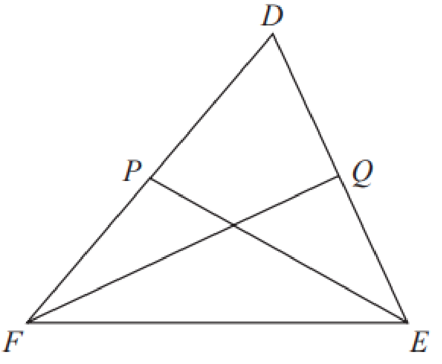
M is the point on *OR* such that $OM : MR = 2 : 3$

Express $\overrightarrow{MT}$ in terms of **a** and **b**.
Give your answer in its simplest form.

....

(Total for Question 20 is 4 marks)

21 *DEF* is a triangle.



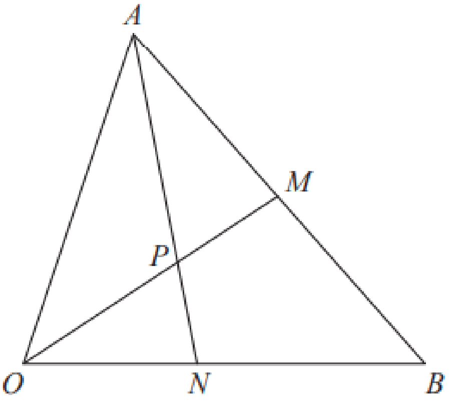
P is the midpoint of *FD*.
Q is the midpoint of *DE*.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$

Use a vector method to prove that *PQ* is parallel to *FE*.

(Total for Question 21 is 4 marks)

21



OAB is a triangle.
OPM and *APN* are straight lines.
M is the midpoint of *AB*.

$\overrightarrow{OA} = \mathbf{a}$ $\quad$ $\overrightarrow{OB} = \mathbf{b}$

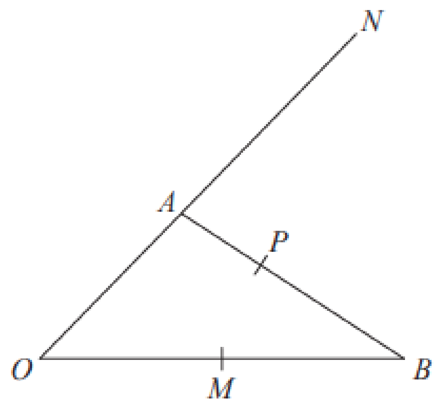
$OP : PM = 3 : 2$

Work out the ratio $ON : NB$

....

(Total for Question 21 is 5 marks)

21



OAN, *OMB* and *APB* are straight lines.
$AN = 2OA$.
M is the midpoint of *OB*.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

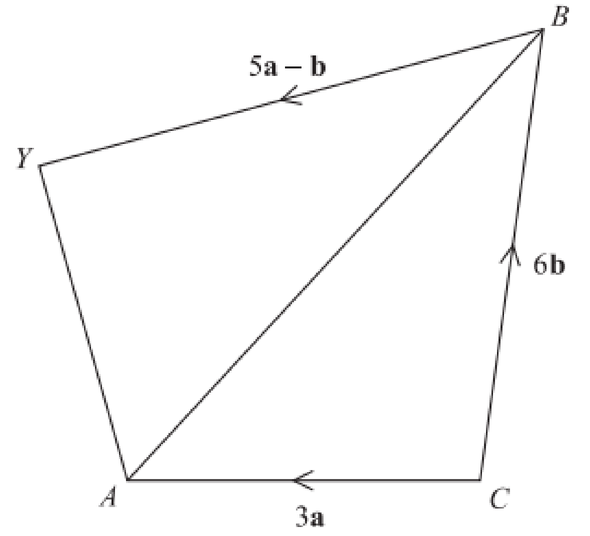
$\overrightarrow{AP} = k\overrightarrow{AB}$ where k is a scalar quantity.

Given that *MPN* is a straight line, find the value of k.

....

(Total for Question 21 is 5 marks)

22



CAYB is a quadrilateral.

$\overrightarrow{CA} = 3\mathbf{a}$
$\overrightarrow{CB} = 6\mathbf{b}$
$\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

X is the point on *AB* such that $AX : XB = 1 : 2$

Prove that $\overrightarrow{CX} = \frac{2}{5}\overrightarrow{CY}$

(Total for Question 22 is 5 marks)

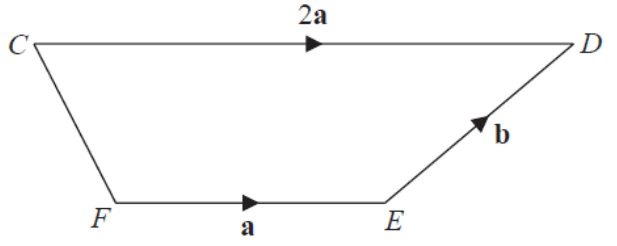
22 Given that the vector $a\begin{pmatrix}2\\6\end{pmatrix} + b\begin{pmatrix}8\\2\end{pmatrix}$ is parallel to the vector $\begin{pmatrix}13\\6\end{pmatrix}$

find an expression for b in terms of a.

....

(Total for Question 22 is 3 marks)

24 *CDEF* is a quadrilateral.



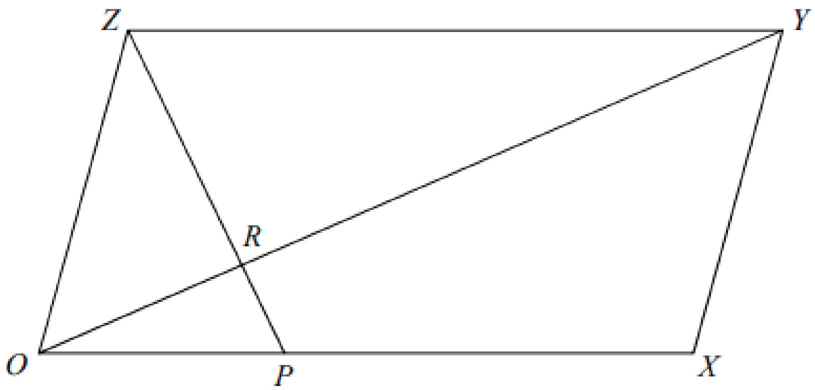
$\overrightarrow{FE} = \mathbf{a}$ $\quad \overrightarrow{ED} = \mathbf{b}$ $\quad \overrightarrow{CD} = 2\mathbf{a}$

The point P is such that CEP is a straight line and that $CE = EP$

Use a vector method to prove that CF is parallel to DP.

(Total for Question 24 is 4 marks)

24 $OXYZ$ is a parallelogram.



$\overrightarrow{OX} = \mathbf{a}$

$\overrightarrow{OY} = \mathbf{b}$

P is the point on OX such that $OP : PX = 1 : 2$
R is the point on OY such that $OR : RY = 1 : 3$

Work out, in its simplest form, the ratio $ZP : ZR$
You must show all your working.

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(Total for Question 24 is 5 marks)